Code No: 131AB

R16

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, January/February- 2024
MATHEMATICS - II
(Common to CE, ME, MCT, MMT, AE, MIE, MSNT)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) If $f(t) = \begin{cases} t-1, & for\ 1 \le t \le 2 \\ 0, & otherwise \end{cases}$ then find $L\{f(t)\}$. [2]

b) If L{f(t)} = $\dfrac{e^{-2s}}{(s^2+1)(s^2+4)}$, then find $f(t)$? [3]

c) Find $\int_0^{\pi/2} \sqrt{\cot\theta d\theta}$. [2]

d) Find $\int_0^1 \left(\log\left(\frac{1}{x}\right)\right)^{1/3} dx$. [3]

e) Write the equivalent integral $\int_0^{2a}\int_0^{\sqrt{x(x-2a)}} dx.dy$ in polar coordinates. [2]

f) Find the double integral of $f(x,y) = \dfrac{x}{y}$ over the region in the first quadrant bounded by the lines $y = x, y = 3x,\ x = 2\ and\ x = 3$. [3]

g) If $\overline{A} = 2x^2\overline{i} - 3yz\overline{j} + xz^2\overline{k}$ then $\nabla.A$. [2]

h) If F and G are vector point functions then, find the expression of $\nabla(\overline{F}\bullet\overline{G})$. [3]

i) Given $F(t) = (5t^2-3t)i + 6t^3 j - 7tk$, evaluate $\int\limits_{t=2}^{t=4} F(t)dt$. [2]

j) Find the value of $\int \nabla(x+y-z)dR$ from (0, 1, -1) to (1, 2, 0)? [3]

PART - B

(50 Marks)

2.a) i) Find the Laplace transform of $f(t) = t\int_0^t \dfrac{e^{-t}\sin t}{t}dt$.

ii) Find the inverse Laplace transform to $\dfrac{se^{-2s}}{s^2+2s+2}$.

b) Find the Laplace transform of f(t) =t/T for 0 ≤ t ≤ T and f(t) is periodic with period T. [5+5]

OR

3. Solve $\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=\sin t, y(0)=0, y'(0)=0$ using Laplace transform. [10]

4.a) i) Evaluate $\int_0^{\pi}\log(1+a.\cos x)dx$ ii) Evaluate $\int_0^{\pi/2}\sqrt{\tan x}dx$.

b) Show that $\beta(p,q)=\int_0^1\frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}dx$. [5+5]

OR

5. Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of gamma functions. Hence evaluate $\int_0^1\frac{1}{\sqrt{1-x^n}}dx$.

[10]

6.a) Find the area lying inside the cardioid $r=a(1+\cos\theta)$ and outside the circle $r=a$.

b) Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$.

[5+5]

OR

7.a) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

b) Using double integration, find the center of gravity of a lamina in the shape of the quadrant of the curve $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$, the density being $\rho=kxy$, where k is a constant. [5+5]

8.a) Prove the vector identity $\nabla\times(\nabla\times\overline{V})=\nabla(\nabla\bullet\overline{V})-\nabla^2\overline{V}$.

b) Find whether $\overline{V}=xyz(yz\bar{i}+xz\bar{j}+xy\bar{k})$ is irrotational, if so find the scalar potential for the vector field $\boldsymbol{V}$? [5+5]

OR

9.a) Prove the vector identity $\nabla.(\overline{F}\times\overline{G})=\overline{G}.(\nabla\times\overline{F})-\overline{F}.(\nabla\times\overline{G})$

b) Show that $\nabla^2(r^n)=n(n+1)r^{n-2}$. [5+5]

10.a) Verify Green's theorem $\oint_c\left[\left(3x^2-8y^2\right)dx+\left(4y-6xy\right)dy\right]$ where C is the closed curve bounded by x=0, y=0 and x + y =1.

b) Verify Gauss divergent theorem for $\overline{f}=x^2\bar{i}+y^2\bar{j}+z^2\bar{k}$ over the region bounded by the cylinder $x^2+y^2=1$ and the planes z = 0 and z = 1. [5+5]

OR

11.a) Show that the vector $\overline{F}=2xy\bar{i}+(x^2+2yz)\bar{j}+(y^2+1)\bar{k}$ is irrotational and find the scalar potential.

b) If $\overline{F}=(x+y+az)\bar{i}+(bx+2y-z)\bar{j}+(x+cy+2z)\bar{k}$, find a, b, c such that $curl\overline{F}=0$, then find ϕ such that $\overline{F}=\nabla\phi$. [5+5]

---ooOoo---